Q1				
(i)	$L = \frac{1}{\sigma_1\sqrt{2\pi}} e^{-\frac{(W_1-\mu)^2}{2\sigma_1^2}} \cdot \frac{1}{\sigma_2\sqrt{2\pi}} e^{-\frac{(W_2-\mu)^2}{2\sigma_2^2}}$	M1 M1 A1	Product form. Two Normal terms. Fully correct.	
	$\ln L = \text{const} - \frac{1}{2\sigma_1^2}(W_1-\mu)^2 - \frac{1}{2\sigma_2^2}(W_2-\mu)^2$	M1 A1		
	$\frac{d\ln L}{d\mu} = \frac{2}{2\sigma_1^2}(W_1-\mu) + \frac{2}{2\sigma_2^2}(W_2-\mu)$	M1 A1	Differentiate w.r.t. μ.	
	$= 0 \Rightarrow \sigma_2^2 W_1 - \sigma_2^2\mu + \sigma_1^2 W_2 - \sigma_1^2\mu = 0$	A1		
	$\Rightarrow \hat{\mu} = \frac{\sigma_2^2 W_1 + \sigma_1^2 W_2}{\sigma_1^2 + \sigma_2^2}$	A1	BEWARE PRINTED ANSWER.	
	Check this is a maximum.	M1		
	E.g. $\frac{d^2 \ln L}{d\mu^2} = -\frac{1}{\sigma_1^2} - \frac{1}{\sigma_2^2} < 0$	A1		11
(ii)	$E(\hat{\mu}) = \frac{\sigma_2^2\mu + \sigma_1^2\mu}{\sigma_1^2 + \sigma_2^2} = \mu$	M1		
	$\therefore$ unbiased.	A1		2
(iii)	$\text{Var}(\hat{\mu}) = \left(\frac{1}{\sigma_1^2 + \sigma_1^2}\right)^2 \cdot (\sigma_2^4\sigma_1^2 + \sigma_1^4\sigma_2^2)$	B1 B1	First factor. Second factor.	
	$= \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$		Simplification not required at this point.	2
(iv)	$T = \frac{1}{2}(W_1 + W_2)$			
	$\text{Var}(T) = \frac{1}{4}(\sigma_1^2 + \sigma_2^2)$	B1		
	Relative efficiency (y) $= \frac{\text{Var}(\hat{\mu})}{\text{Var}(T)}$	M1 M1	Any attempt to compare variances. If correct.	
	$= \frac{\sigma_2^4\sigma_1^2 + \sigma_1^4\sigma_2^2}{(\sigma_1^2 + \sigma_2^2)^2} \cdot \frac{4}{\sigma_1^2 + \sigma_2^2}$	A1		
	$= \frac{4\sigma_1^2\sigma_2^2}{(\sigma_1^2 + \sigma_2^2)^2}$	A1	BEWARE PRINTED ANSWER.	5
(v)	E.g. consider $\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2 = (\sigma_1 - \sigma_2)^2 \geq 0$	M1		
	$\therefore$ Denominator ≥ numerator, $\therefore$ fraction ≤ 1	E1		
	[Both $\hat{\mu}$ and T are unbiased,] $\hat{\mu}$ has smaller variance than T and is therefore better.	E1 E1		4
				24

Q2	$\mathrm{f}(x)=\dfrac{\lambda^{k+1}x^k\mathrm{e}^{-\lambda x}}{k!}$, $[x>0\quad(\lambda>0,\ k \text{ integer} \geq 0)]$ Given: $\int_0^\infty u^m \mathrm{e}^{-u}\mathrm{d}u = m!$			
(i)	$\mathrm{M}_X(\theta)=\mathrm{E}[\mathrm{e}^{\theta x}]$ $=\int_0^\infty \dfrac{\lambda^{k+1}}{k!}x^k\mathrm{e}^{-(\lambda-\theta)x}\mathrm{d}x$ Put $(\lambda-\theta)x=u$ $=\dfrac{\lambda^{k+1}}{k!(\lambda-\theta)^{k+1}}\int_0^\infty u^k\mathrm{e}^{-u}\mathrm{d}u$ $=\left(\dfrac{\lambda}{\lambda-\theta}\right)^{k+1}$	M1 M1 M1 A1 A1 A1 A1	For obtaining this expression after substitution. Take out constants. (Dep on subst.) Apply "given": integral = *k*! (Dep on subst.) BEWARE PRINTED ANSWER.	7
(ii)	$Y = X_1 + X_2 + \ldots + X_n$ By convolution theorem:- mgf of Y is $\{\mathrm{M}_X(\theta)\}^n$ i.e. $\left(\dfrac{\lambda}{\lambda-\theta}\right)^{nk+n}$ $\mu=\mathrm{M}'(0)$ $\mathrm{M}'(\theta)=\lambda^{nk+n}(-nk-n)(\lambda-\theta)^{-nk-n-1}(-1)$ $\therefore \mu=\dfrac{nk+n}{\lambda}$ $\sigma^2=\mathrm{M}''(0)-\mu^2$ $\mathrm{M}''(\theta)=(nk+n)\lambda^{nk+n}(-nk-n-1)(\lambda-\theta)^{-nk-n-2}(-1)$ $\therefore \mathrm{M}''(0)=(nk+n)(nk+n+1)/\lambda^2$ $\therefore \sigma^2=\dfrac{(nk+n)(nk+n+1)}{\lambda^2}-\dfrac{(nk+n)^2}{\lambda^2}$ $=\dfrac{nk+n}{\lambda^2}$	B1 M1 A1 A1 M1 A1 M1 A1		8
(iii)	[Note that $\mathrm{M}_Y(t)$ is of the same functional form as $\mathrm{M}_X(t)$ with $k+1$ replaced by $nk+n$, i.e. k replaced by $nk+n-1$. This must also be true of the pdf.] Pdf of Y is $\dfrac{\lambda^{nk+n}}{(nk+n-1)!}\times y^{nk+n-1}\times \mathrm{e}^{-\lambda y}$ [for $y>0$]	B1 B1 B1	One mark for each factor of the expression. Mark for third factor shown here depends on at least one of the other two earned.	3
(iv)	$\lambda=1$, $k=2$, $n=5$, Exact P(Y > 10) = 0·9165 Use of N(15, 15)	M1 M1	Mean. ft (ii). Variance. ft (ii).	

	P(this > 10) = $P\left(N(0,1) > \frac{10-15}{\sqrt{15}} = -1\cdot 291\right)$ = 0·9017 Reasonably good agreement – CLT working for only small *n*.	A1 A1 E2	c.a.o. c.a.o. (E1, E1) [Or other sensible comments.]	 6
				24

Q3				
(i)	$\bar{x} = 36.48$ $s = 9.6307$ $s^2 = 92.7507$ $\bar{y} = 45.5$ $s = 14.8129$ $s^2 = 219.4218$	B1	If all correct. [No marks for use of s_n which are 9.1365 and 14.1823 respectively.]	
	Assumptions: Normality of <u>both</u> populations	B1		
	equal variances	B1		
	$H_0 : \mu_A = \mu_B$ $H_1 : \mu_A \neq \mu_B$	B1	Do <u>NOT</u> accept $\bar{X} = \bar{Y}$ or similar.	
	Where μ_A, μ_B are the population means.	B1		
	Pooled $s^2 = \dfrac{9 \times 92.7507 + 11 \times 219.4218}{20}$ $= \dfrac{834.756 + 24136.64}{20} = 162.4198$	B1	$= (12.7444)^2$	
	Test statistic is $\dfrac{36.48 - 45.5}{\sqrt{162.4198}\sqrt{\dfrac{1}{10} + \dfrac{1}{12}}}$	M1		
	$= \dfrac{-9.02}{5.4568} = -1.653$	A1		
	Refer to t_{20}.	M1	No ft from here if wrong.	
	Double tailed 5% point is 2·086.	A1	No ft from here if wrong.	
	Not significant.	A1	ft only c's test statistic.	
	No evidence that population mean times differ.	A1	ft only c's test statistic.	12
(ii)	Assumption: Normality of underlying population of <u>differences</u>.	B1		
	$H_0 : \mu_D = 0$ $H_1 : \mu_D > 0$	B1	Do <u>NOT</u> accept $\bar{D} = 0$ or similar.	
	Where μ_D is the population mean of "before – after" differences.	B1	The "<u>direction</u>" of D must be CLEAR. Allow $\mu_A = \mu_B$ etc.	
	Differences are 6.4, 4.4, 3.9, -1.0, 5.6, 8.8, -1.8, 12.1	M1		
	($\bar{x} = 4.8$ $s = 4.6393$)		[A1 can be awarded here if NOT awarded in part (i)]. Use of s_n (=4.3396) is <u>NOT</u> acceptable, even in a denominator of $\dfrac{s_n}{\sqrt{n-1}}$	
	Test statistic is $\dfrac{4.8 - 0}{4.6393 / \sqrt{8}}$	M1		
	=2.92(64)	A1		
	Refer to t_7.	M1	No ft from here if wrong.	
	Single tailed 5% point is 1.895.	A1	No ft from here if wrong.	
	Significant.	A1	ft only c's test statistic.	
	Seems mean is lowered.	A1	ft only c's test statistic.	10
(iii)	The paired comparison in part (ii) eliminates the variability between workers.	E2	(E1, E1)	2
				24

<table>
<tr><td>Q4</td><td></td><td></td><td></td><td></td></tr>
<tr><td>(i)</td><td>Latin square.

Layout such as:

<table>
<tr><td></td><td></td><td colspan="5">Locations</td></tr>
<tr><td></td><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td></td><td>I</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>Surf</td><td>II</td><td>B</td><td>C</td><td>D</td><td>E</td><td>A</td></tr>
<tr><td>-aces</td><td>III</td><td>C</td><td>D</td><td>E</td><td>A</td><td>B</td></tr>
<tr><td></td><td>IV</td><td>D</td><td>E</td><td>A</td><td>B</td><td>C</td></tr>
<tr><td></td><td>V</td><td>E</td><td>A</td><td>B</td><td>C</td><td>D</td></tr>
</table></td><td>B1

B1

B1</td><td>(letters = paints)
Correct rows and columns.

A correct arrangement of letters.
SC. For a description instead of an example allow max 1 out of 2.</td><td>3</td></tr>
<tr><td>(ii)</td><td>$X_{ij} = \mu + \alpha_i + e_{ij}$

μ = population
grand mean for whole experiment.

α_i = population
mean amount by which the i^{th} treatment differs from μ.

e_{ij} are experimental errors
~ ind
N(0, σ^2).</td><td>B1

B1
B1

B1

B1

B1
B1
B1
B1</td><td>

Allow "uncorrelated".
Mean.
Variance.</td><td>9</td></tr>
<tr><td>(iii)</td><td>Totals are: 322, 351, 307, 355, 291
(each from sample of size 5)
Grand total: 1626
"Correction factor" CF = $\frac{1626^2}{25} = 105755.04$

Total SS = 106838 – CF = 1082.96
Between paints SS = $\frac{322^2}{5} + ... + \frac{291^2}{5} - \text{CF}$
= 106368 – CF =612.96
Residual SS (by subtraction) = 1082.96 – 612.96
= 470.00

<table>
<tr><td>Source of variation</td><td>SS</td><td>df</td><td>MS</td></tr>
<tr><td>Between paints</td><td>612.96</td><td>4</td><td>153.24</td></tr>
<tr><td>Residual</td><td>470.00</td><td>20</td><td>23.5</td></tr>
<tr><td>Total</td><td>1082.96</td><td>24</td><td></td></tr>
</table>
MS ratio = $\frac{153.24}{23.5} = 6.52$</td><td>M1
M1

A1

B1
B1
M1

M1
A1</td><td>For correct methods for any two SS.

If each calculated SS is correct.

Degrees of freedom "between paints".
Degrees of freedom "residual".
MS column.

Independent of previous M1.
Dep only on this M1.</td><td></td></tr>
</table>

	Refer to $F_{4, 20}$	M1	No ft if wrong. But allow ft of wrong d.o.f. above.	
	Upper 5% point is 2.87	A1	No ft if wrong.	
	Significant.	A1	ft only c's test statistic and d.o.f.'s.	
	Seems performances of paints are not all the same.	A1	ft only c's test statistic and d.o.f.'s.	12
				24